Memristive circuits
An in-depth analysis of their stochastic dynamics

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1 Introduction

Memristors, resistors with memory, are versatile circuit elements with several applications in computer science, solid state physics, and complex systems.

The resistance value of a memristor varies according to the "history" of some external voltage; the system also remembers the current that has lastly flowed across it, hence storing information about its past states. The tell-tale sign of this behaviour is the typical pinched hysteresis loop for the current vs voltage response (when an sinusoidal driving voltage is applied), that led Leon Chua[1] to coin the expression "If it's pinched it's a memristo".

By design, memristors are a perfect example of analogue computing models, since they incorporate hard disk (memory) and RAM (processing) together in the same unit.[2] This unique design primarily suits its application to topics at the frontier between computer science and neuroscience such as artificial intelligence and neuromorphic computing. As a consequence, the implementation of memristive computing circuits has direct impact onto the development of machine learning algorithms such as *reservoir computing.*

Understanding the dynamics of memristive circuit is also relevant, and compelling, from the point of view of the physics of disordered and non-equilibrium systems. As the equations of motion for the *internal memories* (the state parameter of memristive circuits) are highly non-linear, the dynamics of the system shows interesting collective and emergent behaviours such as swarming, chaos, and self-organization.

The scope of this talk is to present an in-depth analysis of the stochastic dynamics of a circuit of ideal memristors for medium and long times. We will see in which range of parameters the dynamics of a memristive circuit is ergodic, and how entropy production rate is affected by circuit conservation laws, such as Kirchhoff's laws, and the presence of memory. We consider this kind of analysis to be relevant both for the applications to neuromorphic computing as well as to further expand our understanding of crucial aspects of the dynamics of disordered systems. We have chosen to include some form of noise in our analysis (stochastic dynamics) to reproduce real-life scenarios.

2 Ergodicity of memristors circuit

The concept of ergodicity, first introduced by Boltzmann in the 1870s, is pivotal in the framework of classical statistical mechanics as it enables us to relate the microscopic and macroscopic states of a multi-particle system. A direct consequence of the ergodicity hypothesis, put in lay-man terms, is that the dynamics of an ergodidc system is fairly stable under significant changes of initial conditions.

The ergodicity hypothesis is particularly relevant for optimisation processes: if the dynamics of a system is sensitive to its initial configurations, the system may not be able to reach its global

[1]Who firstly inferred their existence using symmetry arguments between non-linear circuit elements, [1]

[2]Memristive computing devices can therefore have higher computational speed, smaller components and lower energy use.

minimum. Understanding whether and how the stochastic dynamics of memristive circuits achieves effective ergodicity is therefore pivotal to their application to reservoir computing.

For this purpose, we have developed a modified version of the *Thirumalai-Mountain (TM) metric*, originally formulated by Thirumalai and Mountain to study the effective ergodicity in fluids and glasses [2, 3]. The modified TM metric (MTMM) takes into account the fact that, differently to glassy systems and fluids, the dynamics of memristive circuits cannot, in general, be accurately described solely by one macroscopic variable (check this). MTMM can be used to measure whether and at which rate a system becomes ergodic.

We have tested the MTMM on the case of linearised equations of motions for the memristors internal memories. In this setting, the internal memories dynamics depends on the external currents and voltages only perturbatively. As expected[3], the MTMM shows that the system (of internal memories) becomes ergodic, by following a power law with exponent $p \sim -1.1$, compatible with the so-called *diffusive regime*.

We have also applied MTMM to the highly non-linear case, for which the internal memories' equations of motion depend non-perturbatively upon external sources (either currents or voltages). In particular we have considered two ranges of external, driving voltages: one range for which the system dynamics becomes chaotic, as shown in [4][4]; one for which the system dynamics is still highly non-linear but non-chaotic. In the second case the MTMM shows that the system dynamics reaches ergodicity following a power law, in a similar fashion to the linear case, but with exponent $p \sim -1.5$; in the first case the system eventually reaches ergodicity, but does not follow a power law trajectory.

3 Entropy production rate

Entropy production is a important tool used in non-equilibrium statistical mechanics to describe irreversability. Unfortunately a rigorous definition is quite challenging to obtain. In recent years promising progress has been made by utilising stochastic processes (as an alternative to conservation equations), see [5].

As previously mentioned, memristors stochastic dynamics falls under the vast category of non-equilibrium statistical mechanics, and it is therefore quite natural to ask the questions: What can we learn about irreversability when we consider a memristive circuit? How would time non-locality (memory) affect entropy production? Furthermore, memristive circuits obey Kirchhoff's laws for conservation of charge and current. We therefore expect to see a signature of these conservation laws in the entropy production rate. In this second part of the talk we will present our results in this respect.

To calculate the entropy production rate we have, for now, considered the linearised versions of the internal memories' equations of motion (the equations depend only pertubatively on external currents and voltages), with some random noise (Wiener process). This is because deriving the entropy production rate requires obtaining the probability density function for the stochastic process (via the Fokker-Plank equation), task exponentially harder for non-linear stochastic processes. The case we have considered is equivalent to n-dimensional linear Langevin equations, for which solving the Fokker-Plank equation and calculating the probability density distribution is fairly straightforward. This has allowed us to calculate the entropy production rate, following a similar derivation as in [6].

References

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[2] D. Thirumalai and R. D. Mountain. Activated dynamics, loss of ergodicity, and transport in supercooled liquids. *Phys. Rev. E*, 47:479–489, Jan 1993.

[3] In this regime most of the non-linear, collective effects are not present. Therefore we expect the system to evolve in an ergodic manner.

[4] In this works the authors have found that, for certain ranges of external voltages, the internal memory dynamics becomes chaotic. The authors have also shown that this behaviour is transient, given that memristors are passive circuit elements.

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