Network effects on self-organisation in congestion games with bounded rational agents

Benjamin Patrick Evans
Centre for Complex Systems
University of Sydney
Sydney, NSW 2006, Australia

Vincent Buskens
Department of Sociology/ICS
Utrecht University
Utrecht, Netherlands

Mikhail Prokopenko
Centre for Complex Systems
University of Sydney
Sydney, NSW 2006, Australia

1 OVERVIEW

Congestion games are common throughout all walks of life. For example, drivers must choose a route subject to traffic conditions. Companies need to select an industry to enter where profitability depends on the number of competitors. In real estate, investors choose a location where competition can affect profitability. In all cases, agents are faced with a decision where the payoffs depend on other agents' decisions. Frequently, such decisions must be made simultaneously and without communication.

Equilibrium solutions to congestion games exist, where agents are perfectly self-organised and instantaneously coordinated. However, multiple questions arise, such as what equilibrium is selected? or are the equilibrium solutions good models of actual human dynamics? In many cases, human play systematically deviates from the predicted equilibrium solutions. Therefore, capturing out-of-equilibrium dynamics is essential, e.g., to explain the endogenous formation of traffic jams on a particular route, or booms and busts in particular industries. Here, rather than enforcing the rationality assumption, we instead focus on the (potentially) out-of-equilibrium dynamics generated from boundedly rational players, who may not play optimally in the sense of Nash, but instead learn and adapt their behaviour over time.

In such games, it is often also assumed that the output of all options, e.g., the profitability, or congestion rate, is known to all agents after each time step (perfect monitoring). However, in many cases, such knowledge is not common (imperfect monitoring); instead, agents know about the outcome of their own choice and only learn about the choices of other agents with whom they have connections (local knowledge). For example, a driver may not know the time taken on alternative routes, or a business may not know the exact profit rates of alternate options. However, agents may have some knowledge based on their connections. In such scenarios, the social network connecting agents is "often found to be a crucial concept in understanding many phenomena" [1], where the topological structure can shape the collective behaviour [2].

In this work, we investigate how the network topology guides agents' decisions and self-organisation of the resulting dynamics in repeated congestion games. Specifically, we explore the convergence, efficiency and volatility of the system. The main findings of this work can be summarised as follows: High clustering in the social network results in excess volatility due to a reduction in the diversity of incoming signals among agents, reducing the ability to self-organise towards a stable outcome. As a result of this increase in overall volatility, the time taken for the agents to converge to a stable solution also increases. In these repeated congestion games, a guiding process that restricts agents' historical decision sharing, therefore, improves the agents' ability to self-organise towards a stable outcome.

2 METHODS

N agents, located at a common origin, are simultaneously faced with choosing an option a amongst a set of options $a \in A$ to arrive at a common destination (visualised in Fig. 1). Each option has a "carrying" capacity X_a and is subject to congestion based on the number of agents also selecting a, at time t, given by $x_{a,t}$ and the capacity X_a. The capacity X_a is assumed to be fixed across t, thus omitting the subscript t. It is also assumed that the *capacity* of each option X_a is common knowledge, but not the *demand* $x_{a,t}$. After choosing an option, the game is repeated at $t+1$, where again, the agent is attempting to choose the option with the maximal payoff U_t.

Corresponding email: benjamin.evans@sydney.edu.au.

Part of this work was conducted while BPE was visiting VB, at the Centre for Complex Systems Studies, Utrecht University, supported by the Swaantje Mondt fund.

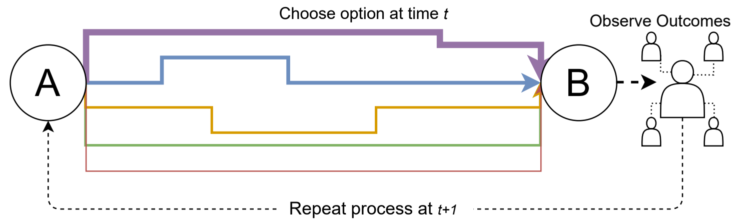

Figure 1: Congestion Game

2.1 Congestion

To implement the effect of "congestion" in such a setting, we utilise the commonly used Bureau of Public Roads (BPR) travel time function, which has also been used in, e.g., [3]. The utility for an agent is then given by the (negative) BPR:

$$U_t[a] = -\underbrace{\tau_a}_{\text{Baseline cost}} \times \underbrace{\left\{1 + \alpha \left(\frac{x_{a,t}}{X_a}\right)^{\lambda}\right\}}_{\text{Congestion component}} \quad (1)$$

where τ_a is the "baseline" utility for an option a without any agents selecting it, which is assumed to be fixed across t. We use the coefficients established in prior work [4]: $\alpha = 0.15$ and $\lambda = 4$. This expression penalises heavily congested routes. For example, a route may take longer the more people who take it, or profitability may diminish with additional competition.

2.2 Decision making

Agents are represented as informationally constrained strategic reasoning agents, following the BRATS approach [5]. BRATS agents attempt to maximise their perceived utility $\hat{U}_t$; however, they are subject to bounds in their information processing abilities based on resource parameter β, following the QH model [6]. Agents have heterogeneous β's sampled uniformly from the range $0 < \beta < 1$. As agents can not communicate prior to a choice, they do not have the exact decision information $x_{a,t}$ available to compute the utility, so instead, they compute a "perceived" utility $\hat{U}_t[a|\hat{x}_{a,t}]$ based on how they believe other agents will choose $\hat{x}_{a,t}$. At the simplest level of reasoning, $\hat{x}_{a,t}$ is determined by assuming all agents behave the same as they did at $t-1$. That is, a level-0 thinker ($\beta = 0$) will assume their neighbours will choose the same outcome as the previous time step (at the first time step, agents assume each option has $x_{a,0} = 0$), and that these neighbours are representative of the population as a whole, and act accordingly. A level-1 thinker attempts to account for this, and bases their decision on the assumption other agents have the same incoming information, and will perform level-0 reasoning. Recursively, level-k agents reason about a distribution of level-$(k-1)$ thinkers, altering their perceived $\hat{x}_{a,t}$, before "depleting" their reasoning resources.

To study the social network effects on self-organisation, we restrict the local interactions among agents, using the Watts–Strogatz (WS) network [7]. It is well known that the WS network topology represents a wide range of real-world social networks [8]. Most notably, varying the rewiring parameter p from a completely regular lattice $p = 0$ (regularity) to a random network $p = 1$ (disorder) generates small-world networks at a particular point between these limits. One of the crucial network characteristics is the clustering coefficient C which quantifies how clusters can form within the networks through altering the mean degree k and rewiring parameter p. The relationship between k, p and C is visualised in Fig. 2.

We are specifically interested in how network clustering affects the ability of agents in the system to self-organise, as it has been conjectured that the

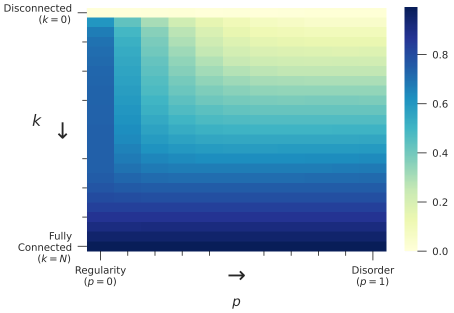

Figure 2: Clustering Coefficient C. Each cell shows the clustering coefficient $C[k, p]$ for the specific values of k and p, for network size $N = 100$ averaged over 20 runs.

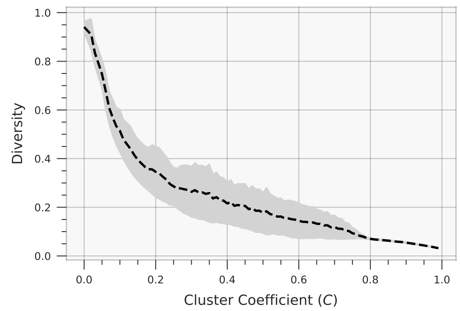

Figure 3: Diversity of signals D. Clustering coefficient values were produced by varying p, k, and diversity calculated from each node outputting sample signals.

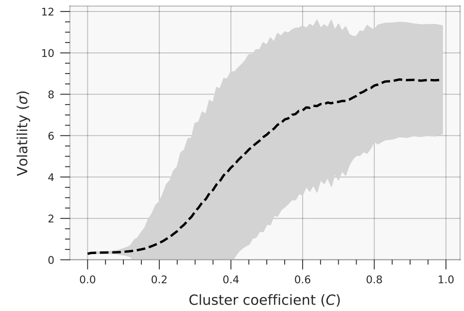

Figure 4: Overall volatility V. Clustering coefficient values were produced by varying p, k.

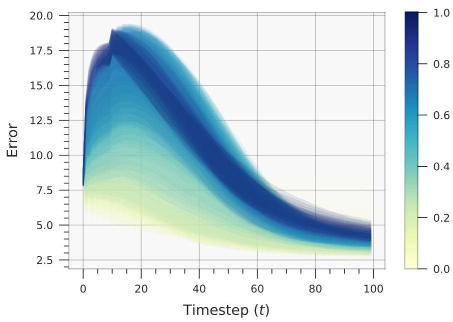

Figure 5: Error rate E_t over time. Clustering coefficients are presented as colours following Fig. 2.

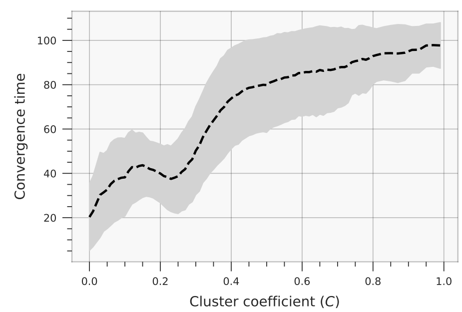

Figure 6: Convergence. The time taken to reach a "stable" setting. Clustering coefficient values were produced by varying p, k.

diversity of agent beliefs is one of the "most important propagation mechanisms" of volatility [5, 9]. To measure diversity D, we use the normalised sum of the pairwise Euclidean distances between the agent's incoming signals. An agent n's incoming signal $\vec{I_n}$ captures the distribution of neighbours selecting each option. The diversity D is then measured as:

$$D = \frac{1}{\max_D} \left(\sum_{i \in N} \sum_{j \in N} d(\vec{I_i}, \vec{I_j}) \right) \tag{2}$$

where d is the Euclidean distance function, and $\max_D$ is the maximum theoretical Euclidean distance.

The relationship between C and D is visualised in Fig. 3. As the clustering coefficient increases, the diversity of the agents' decision signals reduces markedly, as agents have similar neighbours (and thus, similar input signals).

3 RESULTS

This section analyses the results of our dynamic model. Specifically, the convergence, efficiency, and volatility, are considered and related to the bounded rational nature of the agents and their incoming decision signals based on their social network topology.

3.1 Volatility

The volatility of the system is based on the fractional change in the number of agents choosing an option at each timestep, denoted by $\vec{r_a}$ (i.e., a vector of length T, where each element $r_{a,t}$ is the volatility at time t). The overall volatility V is then defined as the mean standard deviation of these changes:

$$V = \frac{1}{\#A} \sum_{a \in A} \sigma(\vec{r_a}) \tag{3}$$

where $\#A$ is the cardinality of A. The network topology is found to have a drastic effect on the self-organisation of the agents, with the clustering coefficient affecting the overall system volatility V. This effect is shown in Fig. 4, where as the clustering in the network increases, the overall volatility also increases. This dependency arises due to the reduction in the heterogeneity of decision signals, underlying "herd"-like behaviour. As the similarity of agent reasoning increases (through similarity of demand estimation), the heterogeneity of the population reduces, and thus, more significant overall volatility is likely, resulting in reduced efficiency of the system.

3.2 Efficiency and Convergence

Efficiency is expressed via the resulting error rate (Fig. 5), as governed by:

$$E_t = \frac{1}{\#A} \sum_{a \in A} |X_a - x_{a,t}| \tag{4}$$

i.e., the mean absolute error between the carrying capacities X_a and the resulting selecting rates $x_{a,t}$.

Convergence is expressed as the time taken for agents to self-organise to a stable (but not necessarily optimal) outcome (Fig. 6). The path to convergence (and resulting efficiency) strongly depend on the clustering coefficient. The number of timesteps taken for the agents to converge increases non-linearly with C (Fig. 6). These results show that the diversity of incoming signals is an essential mechanism for guiding the self-organisation process of the agents, as the increased heterogeneity can help with the convergence. This finding concurs with discussions on the El Farol bar problem [10], where the randomisation of agent beliefs was suggested to ensure convergence to desirable macro-outcomes [11].

4 CONCLUSION

This work explored the effects of network topology and bounded rational reasoning on self-organisation in repeated congestion games. The main result is that volatility arises endogenously in response to changes within the network topology. Specifically, increasing clustering was shown to reduce the diversity of agent beliefs, which in turn, increases the overall volatility. Lack of diversity has long been hypothesised to explain the emergence of volatility. By altering the network topology, we quantified how such diversity non-linearly guides the system dynamics in congestion games.

REFERENCES

[1] J. Broere, V. Buskens, J. Weesie, and H. Stoof, “Network effects on coordination in asymmetric games,” *Scientific reports*, vol. 7, no. 1, pp. 1–9, 2017.
[2] D. Kasthurirathna and M. Piraveenan, “Emergence of scale-free characteristics in socio-ecological systems with bounded rationality,” *Scientific reports*, vol. 5, no. 1, pp. 1–16, 2015.
[3] C.-L. Zhao and H.-J. Huang, “Modeling bounded rationality in congestion games with the quantal response equilibrium,” *Procedia-Social and Behavioral Sciences*, vol. 138, pp. 641–648, 2014.
[4] U. S. B. of Public Roads, *Traffic assignment manual for application with a large, high speed computer*, vol. 2. US Department of Commerce, 1964.
[5] B. P. Evans and M. Prokopenko, “Bounded strategic reasoning explains crisis emergence in multi-agent market games,” *arXiv preprint arXiv:2206.05568*, 2022.
[6] B. P. Evans and M. Prokopenko, “Bounded rationality for relaxing best response and mutual consistency: The Quantal Hierarchy model of decision-making,” *arXiv preprint arXiv:2106.15844*, 2021.
[7] D. J. Watts and S. H. Strogatz, “Collective dynamics of ‘small-world’networks,” *nature*, vol. 393, no. 6684, pp. 440–442, 1998.
[8] M. Newman, *Networks.* Oxford university press, 2018.
[9] M. Kurz and M. Motolese, “Endogenous uncertainty and market volatility,” *Economic Theory*, vol. 17, no. 3, pp. 497–544, 2001.
[10] W. B. Arthur, “Inductive reasoning and bounded rationality,” *American Economic Review*, vol. 84, no. 2, pp. 406–411, 1994.
[11] D. Challet, M. Marsili, and G. Ottino, “Shedding light on El Farol,” *Physica A: Statistical Mechanics and Its Applications*, vol. 332, pp. 469–482, 2004.