Mean-field Estimation of Fisher Information in Random Boolean Networks

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Previous studies of Fisher information [3, 5] have shown that Fisher information is a useful measure for capturing second order phase transitions. The study [5] on random Boolean networks was limited by network size due to its simulation based approach in which individual node trajectories were generated, while [3] related Fisher information to the rate of change of order parameters explicitly. Here, we study Fisher information in *infinite* random Boolean networks by developing a suitable mean field framework.

We begin by highlighting a mean field model proposed in [4] which considers a distribution of local node biases. This model provides a compromise between loss of individual node identity, and preservation of information theoretic quantities. We show, for example, that entropy estimates are consistent with finite network studies [2].

A novel approach is required to estimate Fisher information under this mean field model, as we no longer have access to individual node trajectories (and the network is infinitely large). We outline a rule-based approach to estimating the Fisher information of the network state with respect to the average connectivity, and show empirically that the measure is maximized at the well known phase transition [1]. This finding supports [5], and provides significant evidence that the Fisher information is indeed a useful measure for studying second order phase transitions.

The same approach is used to estimate the Kullback-Leibler divergence between network state distributions as we vary the average connectivity. Under weak regularity conditions this measure is approximately proportional to the Fisher information. Indeed, we show that this measure demonstrates the same qualitative behavior as the Fisher information, providing further confidence in our primary claim as no finite difference approximations are required to compute the Kullback-Leibler divergence.

References

[1] B. Derrida, Y. Pomeau, '*Random networks of automata: a simple annealed approximation*', Europhys. Lett. Vol. 1, pp. 45-49, 1986.

[2] J. T. Lizier, M. Prokopenko and A. Y. Zomaya, '*The information dynamics of phase transitions in random Boolean networks*', Proc ALife XI, pp. 374-381, 2008.

[3] M. Prokopenko, J. T. Lizier, O. Obst, X. R. Wang, '*Relating Fisher information to order parameters*', Physical Review E, Vol. 84, pp. 041116, 2011.

[4] A. S. Ribeiro, S. A. Kauffman, J. Lloyd-Price, B. Samuelsson, J. E. S. Socolar, '*Mutual information in random Boolean models of regulatory networks*', Physical Review E, Vol. 77, pp. 011901-10, 2008.

[5] X. R. Wang, J. T. Lizier, M. Prokopenko, '*Fisher information at the edge of chaos in random Boolean networks*', Artificial Life, Vol. 17, No. 4, pp. 315-329, 2011.