Modeling and analyzing large swarms with covert leaders*

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In this paper, we report on efforts to analyze general models of large swarms with *covert leaders*. We focus our efforts on behavior driven by three-zone swarming. In three-zone swarming behavior, individual behavior is driven by the position and orientation of neighboring individuals in each of three concentric zones. An individual will try to move away from others in the innermost zone of repulsion. An individual will try to align itself with others in the central zone of orientation. Finally, an individual will try to move toward individuals in the outermost zone of attraction. Individual behavior is the weighted contribution from the three influences. The fundamental purpose of this research is to understand how interactions between individuals are mapped to the motion and properties of the entire swarm, and how the presence of covert leaders within the swarm changes the mapping.

A leader is an individual guided by additional information that ordinary individuals, *followers*, do not possess. A *covert leader* is a leader that is treated no differently from a follower. More precisely, all swarm influences must be functions of the sum of the covert leaders and followers taken together because it is not possible to distinguish one from the other. We will understand the dynamics of large swarms by treating the continuum limit of the underlying model. In this limit, individuals are represented as a density which is a function of space and time. Similarly, the velocities of discrete individuals are represented by a velocity function. The dynamics of the swarm is described by a system of coupled partial differential equations capturing necessary conservation principles and the local interactions (i.e. behavior) between neighboring members of the swarm.

Using this concise mathematical approach, one can search for stable structures in large swarms and determine how they depend upon size and influence of each of the zones. Earlier results explain the emergence of specific axisymmetric and non-axisymmetric structures in swarms without leaders. In addition, we could explain how smaller compact structures are preferred over larger structures given certain parameter combinations. These results also apply to traditional leadership models in which leader behavior is a linear combination of swarm interactions and response to additional information. However, in these regimes, leadership only affects where the swarm goes, but not how the ensemble behaviors. We present a new nonlinear model in which a leader will respond more strongly to additional information when the swarm is less dense, meaning there are fewer individuals with which to interact. Conversely, leaders in dense regions behave more like followers. Since the response is slightly nonlinear, new stability criteria and new structures emerge as a function of swarming parameters.

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