Transfer entropy in continuous time: from computational neuroscience to information thermodynamics

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Transfer entropy is now a well established measure of directed information flow [1] with many diverse applications [2, 3, 4]. However, as it was initially formulated, it concerns discrete time systems. We extend the concept to continuous time systems which form the basis for a great deal of physically occuring models [5]. By doing so we can apply such a formalism to continuous time spiking processes, essential to understanding neuron activity in computational neuroscience [6, 7, 8, 9, 10, 11, 12, 13, 14] and can also implicate transfer entropy within modern formalisms of thermodynamics which deal in ensembles of continuous path functions [15, 16, 17, 18, 19, 20]. We then utilise key notions from thermodynamics, such as time reversal [21], in the construction of informational dynamical quantities with the aim of understanding the ever strengthening connection between thermodynamics and information [22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32] in order to understand the transfer of information physically and to make progress towards a unifying information thermodynamics which has applications as diverse as designing molecular machines of the future [16] through to fundamental concepts such as the arrow of time [33, 34, 35, 36, 37].

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